THE KING'S SCHOOL, CANTERBURY

SCHOLARSHIP ENTRANCE EXAMINATION

February 2013

MATHEMATICS 1

Time: 45 minutes (plus reading time)

Use the reading time wisely; gain an overview of the paper and start to think of how you will answer the questions.

Do as many questions as you can (clearly numbered) on the lined paper provided. Clearly name each sheet used. You are encouraged to attempt these questions in order.

The questions are not of equal length or mark allocation. Make sure you avoid spending too much time on any one question; don't get bogged down! Move on quickly if you get stuck. The paper is quite long; you are not necessarily expected to finish everything.

Some of the later questions are more difficult, but not necessarily longer. Some questions are designed to test your ability to work with unfamiliar ideas, or familiar ones with a twist. Don't give up!

*You are expected to use a calculator where appropriate, but also you must show **full and clear working**, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers. In fact, merely writing down an answer might score very few marks.*

Complete solutions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.

This paper has ten questions.

1 Last year, I read somewhere that the world was going to end the day after 20.12.2012 (it didn't).

When is the next date which repeats in this pattern *ab.cd.abcd* ?

[Note that *a* and *d* do not necessarily have to be the same, although they are on this date.]

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2 (a) Write 2013 as a product of its prime factors.

(b) How many digits does the number $2^{81} \times 5^{79} \times 10^{21}$ have? [Show your reasoning carefully.]

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3 Solve the following for *x*:

(a) $7 - \frac{x}{8} = 11$

(b) $2(3 + x) - (4 - x) = 29$

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4 Eighty of Pythagoras' μ-burgers (supposed to be made with beef) were tested and variously some of them were found to contain horse, zebra and donkey meat.

40	contain horse [n.b. these do not mean horse ONLY, etc.]
35	contain zebra
33	contain donkey
12	contain horse and zebra
16	contain zebra and donkey
14	contain horse and donkey
5	contain horse, zebra and donkey

All of them contain some beef, but how many burgers did not contain any of these other meats?

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5 Wikipedia compares the described length of the marathon running race using both imperial and metric units as follows:

The 42.195 km and 26 miles 385 yards distances are identical to within half an inch.

Work out whether this is true or false. Show all working carefully. Use any of the below as needed.

Imperial to	Metric
1 inch	2.54 cm
1 foot [ft] = 12 inches	0.3048 m
1 yard [yd] = 3 ft	0.9144 m
1 mile = 1760 yd	1.6093 km

Metric to	Imperial
1 millimetre [mm]	0.03937 in
1 centimetre [cm] =10 mm	0.3937 in
1 metre [m] = 100 cm	1.0936 yd
1 kilometre [km] = 1000 m	0.6214 mile

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6 Here is the 7mm frog *Paedophryne amauensis*, recently claimed to be the world's smallest vertebrate.

Suppose the frog can jump left or right (along a straight line) by a distance which doubles every time, starting 1mm, 2mm, 4mm, and so its n^{th} jump is 2^{n-1} mm.

Can the frog get back to its starting point by a suitable choice of direction each time?
If you think yes, show how this may be done.
If you think no, explain why it is impossible.

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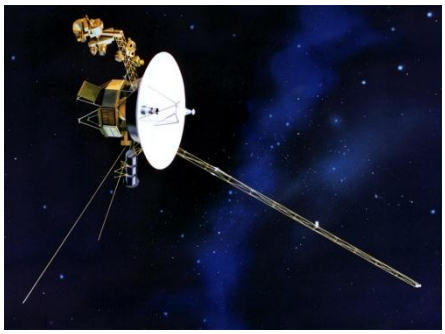
7 (a) The Voyager 1 spacecraft has travelled approximately 18 billion km from Earth since its launch in September 1977 and is soon to leave the Solar System.



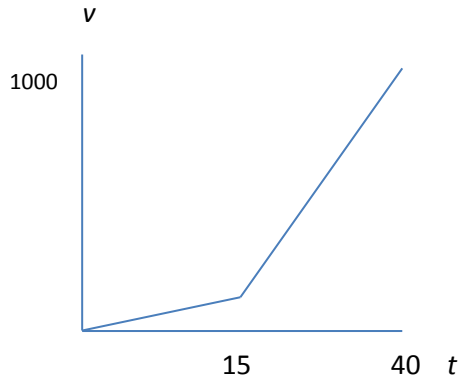
Work out an approximate average speed for Voyager 1 during this time, showing all your working, and giving your answer in appropriate units.

[Don't worry about leap years.]

(b) This year the Bloodhound project hopes to break the land speed record by exceeding 1000 mph in a specially designed car.

Bloodhound will take 15 seconds to reach 100mph but it will then take just 25 seconds to go from 100mph to 1,000mph.

Imagine that in each of these parts the acceleration is constant, so a travel graph of the journey might look like this:



What would be the average speed during this period?
Be careful with the units in your method and answer.

[Hint: the area under the graph helps find the total distance travelled.]

8 After the scholarship mathematics examinations the candidates are surveyed.

70% enjoyed paper 1 and 80% enjoyed paper 2.

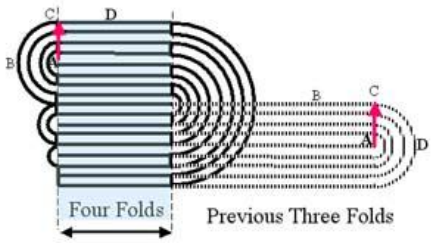
(a) What is the maximum for the percentage of candidates who liked both papers?

(b) Find the minimum for the percentage who liked both papers.

9 The ratio of Dylan's age to Alfred's age now is 3 : 1. Five years ago the ratio was 4 : 1.

In how many years' time will the ratio be 2 : 1?

10 In 2002 Britney Gallivan, a US high-school student, solved the "paper-folding problem" and derived a formula for a method of folding a long sheet of paper multiple times, an intermediate stage of which looks like this:



$$L = \frac{\pi t}{6}(2^n + 4)(2^n - 1)$$

[You will need the power button on your calculator here]

where *L* is the minimum possible length of the paper, *t* is paper thickness, and *n* is the number of folds. (*L* and *t* are in the same units).

(a) Britney made 12 folds in paper of thickness 0.1 mm (0.0001 m). Use your calculator to work out how long the piece of paper needs to be.

(b) [hard] The ENCODE DNA genetic data project is so huge that we would need a 30 km long piece of paper to print it all out. How many folds of a 0.1 mm thickness sheet of 30 km length would we be able to do this time? [Find n to the nearest whole number.]

END OF PAPER